Capacitors
Electricity: ~~Current, Resistance and Power: Ohms Law~~

Name: ____________________ Period: 7th Date: ________

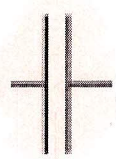
CAPACITANCE

A capacitor is a device for storing charge, made up of two parallel plates with a space between them.

The plates have an equal and opposite charge on them, creating a potential difference between the plates.

A capacitor can be made of conductors of any shape, but the **parallel-plate capacitor** is the most common kind.

In circuit diagrams, a capacitor is represented by two equal parallel lines.



FORMULAS

For any capacitor, the ratio of the charge to the potential difference is called the **capacitance**, C:

$$C = \frac{Q}{V}$$

Where:

C= Capacitance [$\frac{C}{V}$, Farad, F]

Q= Charge [C]

V= Voltage [V]

The unit of capacitance is the **farad** (F). One farad is equal to one coulomb per volt.

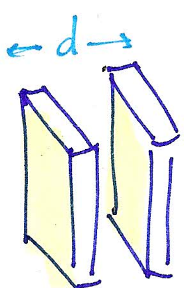
Most capacitors have very small capacitances, which are usually given in microfarads, where $1\ \mu F = 10^{-6}$ F.

$1 nF = 10^{-9} F$

For a parallel-plate capacitor, C is directly proportional to the area of the plates, A, and inversely proportional to the distance between them, d.

Where:

$$C = \varepsilon_o \frac{A}{d}$$



C=Capacitance [C/V= Farad F]

ε_0=Vacuum permittivity= $8.85\times10^{-12} C^2/Nm^2$

A=Plate's surface area [m^2]

d=distance between plates [m]

That is, if the area of the plates is doubled, the capacitance is doubled, and if the distance between the plates is doubled, the capacitance is halved.

The proportionality constant between C and A/d is ε_0, called the permittivity of free space, which is related to Coulomb's constant. $\varepsilon_0 = 8.85\times10^{-12} C^2/Nm^2$

Example: A 10 nanofarad parallel plate capacitor holds a charge of magnitude 50µC on each plate.

a. What is the potential difference between the plates? $C = \frac{Q}{V}$ or $C = \varepsilon_0 \frac{A}{d}$

b. If the plates are separated by a distance of 0.2mm, what is the area of the plate?

$C = 10nF = 10 \times 10^{-9} F$

$Q = 50\mu C = 50 \times 10^{-6} C$

a) $C = \frac{Q}{V}$

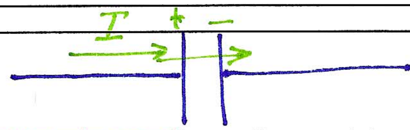
$V = \frac{Q}{C} = \frac{50 \times 10^{-6}}{10 \times 10^{-9}} = 5 \times 10^{5} V$

b) $0.2mm \cdot \frac{1 \, m}{1000 \, mm} = 2 \times 10^{-4} m$

$C = \varepsilon_0 \frac{A}{d}$

$A = \frac{C \cdot d}{\varepsilon_0} = \frac{(10 \times 10^{-9})(2 \times 10^{-4})}{8.85 \times 10^{-12}} = 0.22 m^2$

ENERGY IN CAPATITORS



To move a small amount of negative charge from the positive plate to the negative plate of a capacitor, an external agent must do work. This work is the origin of the energy stored by the capacitor.

If the plates have a charge of magnitude q, the potential difference is $\Delta V = \frac{\Delta Q}{C}$. If $q = 0$, and work is done to add charge until $q = Q$, the total work required is:

$$U = \frac{1}{2} Q \Delta V = \frac{1}{2} C V^2$$

Where:

U=Electric Potential Energy [J]

Q=Charge [C]

ΔV=Voltage [V]

C=Capacitance [C/V= Farad F]

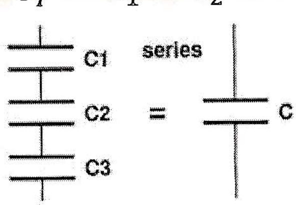
CAPACITORS IN CIRCUITS

Like resistors, capacitors can be arranged in series or in parallel. The rule for adding capacitance is the reverse of adding resistance:

Capacitors in series add like resistors in parallel, and capacitors in parallel add like resistors in series.

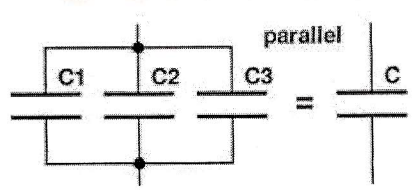
CAPACITORS IN SERIES

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

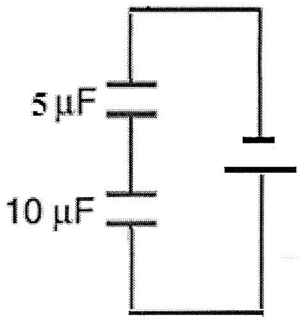


CAPACITORS IN PARALLEL

$$C_T = C_1 + C_2 + \cdots$$



Example: Find the equivalent capacitance for the following circuit.



$$\frac{1}{C_T} = \frac{1}{5} + \frac{1}{10}$$

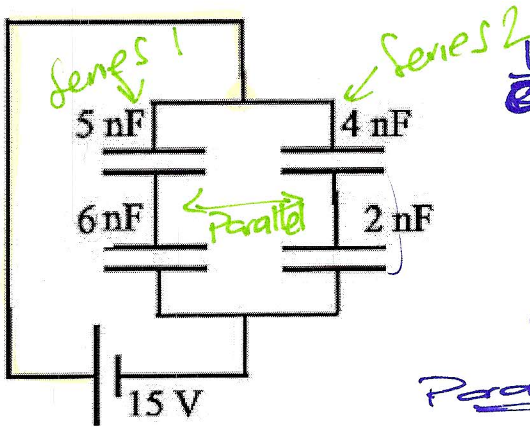
$$\frac{1}{C_T} = \frac{2+1}{10}$$

$$\frac{1}{C_T} = \frac{3}{10}$$

$$C_T = \frac{10}{3} = \boxed{3.33\,\mu F} \quad \text{vs} \quad 3.33\ F$$

0.00000333 F

Example: Find the equivalent capacitance for the following circuit.



Series 1

$$\frac{1}{C_T} = \frac{1}{5} + \frac{1}{6}$$

$$\frac{1}{C} = \frac{6+5}{30}$$

$$\frac{1}{C} = \frac{11}{30}$$

$$C = \frac{30}{11} = 2.73\,nF$$

Series 2

$$\frac{1}{C_T} = \frac{1}{4} + \frac{1}{2}$$

$$\frac{1}{C_T} = \frac{1+2}{4}$$

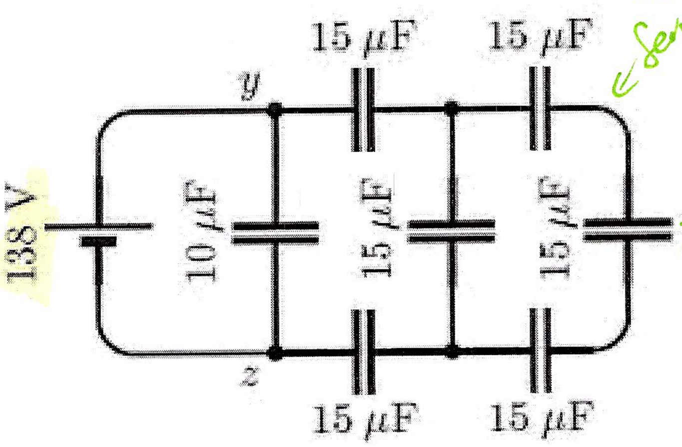
$$\frac{1}{C_T} = \frac{3}{4}$$

$$C_T = \frac{4}{3} = 1.33\,nF$$

Parallel

$$C_T = 2.73\,nF + 1.33\,nF = \underline{4.06\,nF}$$

Example: Find the equivalent capacitance for the following combined circuit.



① $\frac{1}{C} = \frac{1}{15} + \frac{1}{15} + \frac{1}{15}$

$$\frac{1}{C} = \frac{3}{15}$$

$$C = 15/3 = \underline{5}$$

② $5 + 15 = 20$

③ $\frac{1}{C} = \frac{1}{15} + \frac{1}{15} + \frac{1}{20}$

$$\frac{1}{C} = \frac{4+4+3}{60}$$

$$\frac{1}{C_T} = \frac{11}{60}$$

$$C_T = 60/11 = 5.45$$

④ $5.45 + 10$

$C_T = 15.45\,\mu F$